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Development of the Plastic Zone at the Crack Tip Under Cyclic Loading in the Middle-cracked Tension Specimen

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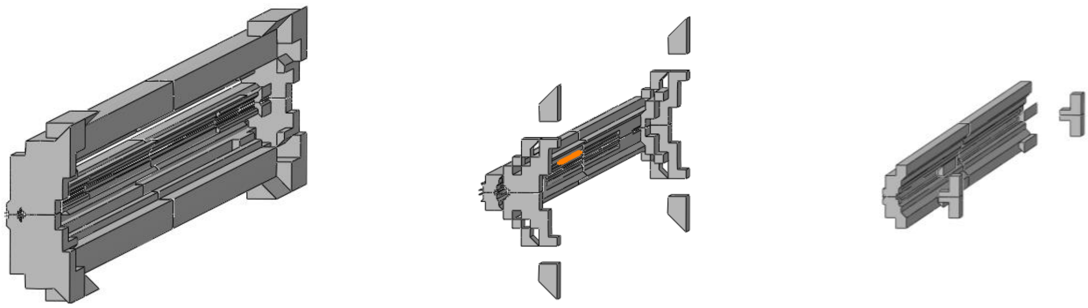
Abstract. The paper deals with the development of the plastic zone size during cyclic loading. The analysis was carried out using finite element method. The three-dimensional model of M(T) test specimen with central crack was discretized by hexahedral elements. In the area of crack tip the mesh was refined. The elastic/perfectly-plastic material with isotropic hardening was employed. Cyclic loading with coefficient of the cycle asymmetry $R = 0$ was applied up to 100 cycles. The plastic zone area in cross-sections and the size in the crack plane was monitored. The cyclic plastic zone size remains approximately constant in the centre of the specimen, where plane strain prevails. Whereas near the free sides, the size is reduced with increasing number of cycles and tends to form the shape and size of plane strain. The development of the cyclic plastic zone in upward part of the cycle can be described by interpolation function in dependence on the monotonic plastic zone.

Introduction

Loading the part with a crack leads to developing the plastic zone at the crack tip in most metal engineering materials. The plastic zone increases in hollow material according to power law. At the sides of the body, the crack tip is in the conditions of plane stress, while toward to the centre, the condition of plane strain prevails, provided the small plastic zone size in comparison with the characteristic dimension of the body. The condition of plane strain is often introduced in the form of the ratio $r_p/B = 0.0637$, where B denotes the characteristic dimension of the body and r_p is the plastic zone size [1]. The FE analysis is widely applied when studying three-dimensional part. The plastic zone and the strain constraint at the crack tip during monotonic loading were studied on laboratory specimens in [2]. The constraint factor α was analysed in order to describe the out-of-plane constraint as well. The plastic zone under cyclic loading is analysed in this work for the ratio of $B/r_{p\,\max} = 79$ (in the reversed form for $r_p/B = 0.0127$) using the same FE-model of M(T) specimen. The ratio is expressed at the maximum load level of the cycle. This indicates that the material at the crack front is mostly in the plane strain conditions. The dimensions of the specimen were 100x80x20 mm with half crack length of 20 mm. The material was described as ideally elasto-plastic with isotropic hardening. The characteristic mesh size at the crack tip was 0.003 mm and the hexahedral elements with reduced integration were used. The mesh size was increased in locations away from the crack tip in order to sustain the analysis price reasonable. Six layers of elements gradually tightening to the sides were used in 1/8 model by applying the symmetry planes. The pre/post processing and the solution using implicit solver with the step increments of 0.1 was carried out in Abaqus 2017 software.

Development of plastic zone during cyclic loading

Cyclic loading creates the cyclic plastic zone at the crack tip and the residual plastic zone is present even in the unloaded state. The elasto-plastic behaviour and strain hardening results in blunting of the crack tip during cyclic loading and the plastic zone is forced to change. The cyclic loading with the coefficient of cycle asymmetry $R = 0$ is analysed. The shape of the plastic zone is shown for 1st cycle in Fig. 1 a), after 10 cycles in b) and after 100 cycles in c). The reduction of size of plastic zone is clearly visible. The plastic zone under plane stress splits apart by application of increasing number of cycles. Nevertheless, only tightening of the plane stress zone might be the reason and the finite element is not thin enough to describe this change.



a) b) c)

Fig.1 Plastic zone after a) 1 cycle, b) 10 cycles and c) 100 cycles; FE analysis ($B / r_{p\max} = 79$).

The plastic zone size measured in units of area. In Fig. 2 a) there is shown the development of the plastic zone area according to rising load S in the cycle for the 1st, 2nd, 10th and the 100th cycle. The area is measured in the crack plane perpendicular to the crack front. Two planes are evaluated; the centre and the side plane. The considerable reduction of the plastic zone size can be seen after 1st cycle, but the size in the following cycles in plane strain region remains the same for each load level in upward part of the cycle. The data can be fitted by the power law, that is shown in the logarithmic scale as the straight line. During monotonic loading, it can be seen the increase of the size with the power exponent k = 4.3 for both plane stress and plane strain conditions. During cyclic loading, the residual plastic zone is also present at the zero load. By increasing the load, the residual plastic zone decreases and at certain load level S_{open}, the forward cyclic plastic zone starts to form. The development of the cyclic plastic zone should be described from the opening stress S_{open} when the plastically deformed material is not present. The normalized loading $\left(S - S_{open}\right) / \left(S_{\max} - S_{open}\right)$ in the upward part of the cycle is therefore the unit of the x-axis and the power law will form in Eq. 1, where C and k are coefficients.

$$y = C\left(\frac{S - S_{open}}{S_{\max} - S_{open}}\right)^{k} \tag{1}$$

Applying the cyclic loading causes the cyclic plastic zone. It´s development measured in the area was described with the power exponent k = 1.4 for both plane stress and plane strain conditions. The exponent remains almost the same for the 2nd and the following cycles. The plastic zone size in the upward part of the cycle during cyclic loading is therefore almost the same in the studied range of cycle number. The plastic zone tends to stabilize under the plane strain conditions even near the free sides. The residual plastic zone size development according to cycle number is shown in Fig. 2 b). The size of the residual plastic zone increases with applied cycles.

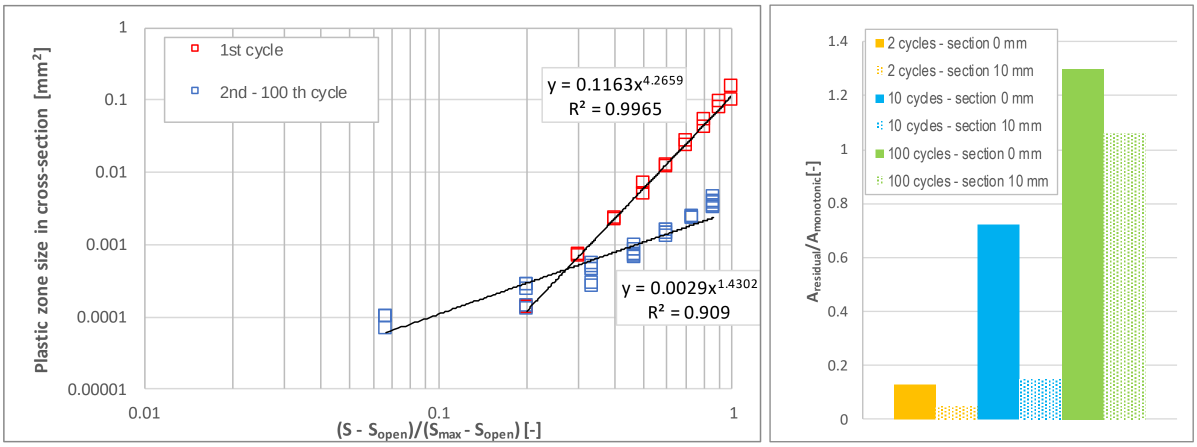


Fig.2 Area of plastic zone after 1 - 100 cycles ($B/r_{p\,\max} = 79$) a) plastic zone size development in upward part of the cycle, b) Residual plastic zone size versus size at the maximum load level in the cycle

The plastic zone size measured as the length in the crack plane. The development of plastic zone size in the crack plane r_p was determined by the similar way as the area of plastic zone. The size r_p is the length measure of the plastic zone size commonly determined analytically according to Irwin [3] or Dugdale [4]. The plastic zone size r_p increases in the 1st cycle on the specimen sides in plane stress with the power exponent k = 3 and in the centre of the specimen in plane strain with the power exponent k = 1.8. The following cycles create the cyclic plastic zone $r_{p\,cyclic}$. Its size is smaller than the size of the monotonic zone from the x-axis rate of 0.5 and increases according to the loading with the power exponent k = 0.7 for both locations; the centre and the sides of the specimen.

During cyclic loading, in upward part of the cycle, the plastic zone size at the specimen's sides is larger and only at the maximum load level of the cycle the size in this location is close to the size reached in the 1st cycle. However, the size at the sides is related to the remote plastically deformed point whereas the continuous plastic zone size is smaller and it is close to the size in the centre under plane strain conditions. The development of plastic zone size for $B/r_{p\,\max} = 79$ during loading is shown in Fig. 2 a).

The residual plastic zone size r_p has almost the same dimension in the centre under plane strain condition for all cycles but slightly increases with rising number of cycles for the sides of the specimen. In comparison with the maximum dimension at the maximum load level in the cycle, the average is around 1.1 except the size after 100 cycles causing the increase of the residual plastic zone at the sides of the specimen with the ratio of 1.6 (see Fig 3 b).

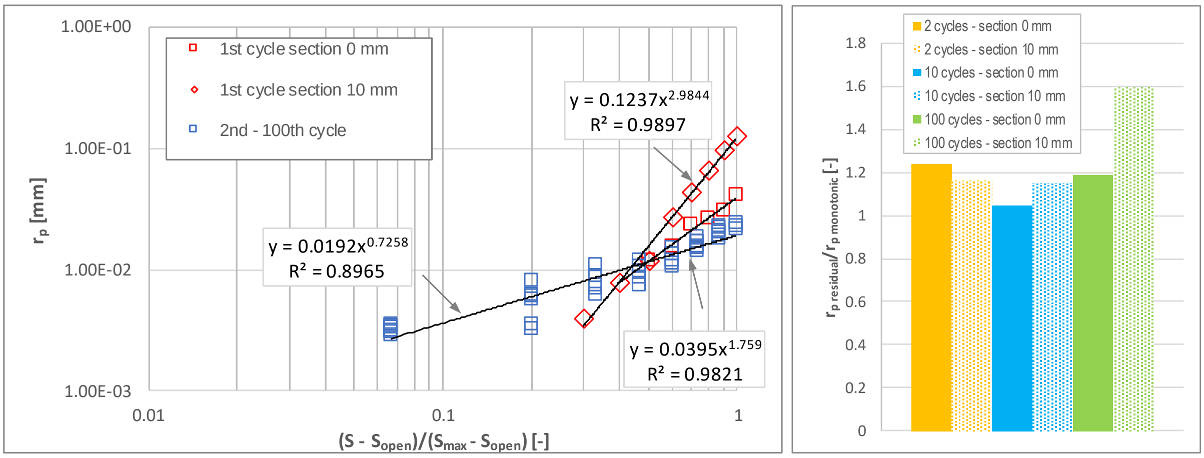


Fig.3 Plastic zone size r_p after 1 - 100 cycles ($B/r_{p\,max} = 79$) a) plastic zone size development in upward part of the cycle, b) Residual plastic zone size versus size at the maximum load level in the cycle

Estimation of the cyclic plastic zone size

In order to predict the size of the cyclic plastic zone $r_{p\,cyclic}$ during cyclic loading, the interpolation function is shown in Fig. 4 together with the values determined by the FE-analysis. Note that the plastic zone size in upward part of the cycle $r_{p\,cyclic}$ appears at the opening stress load level. The opening stress for analysed coefficient of cycle asymmetry $R = 0$ was determined close to the load level $S/S_{\max} \in (0.2, 0.3)$. This value corresponds well with the opening stress of 0.25 determined according to Newman's function for opening stress S_{op} [5]. The limit straight line shown in Fig. 4 estimates the values of $r_{p\,monotonic\,RD}/r_{p\,cyclic}$ from the left side. The subscript *RD* denotes the plane strain conditions.

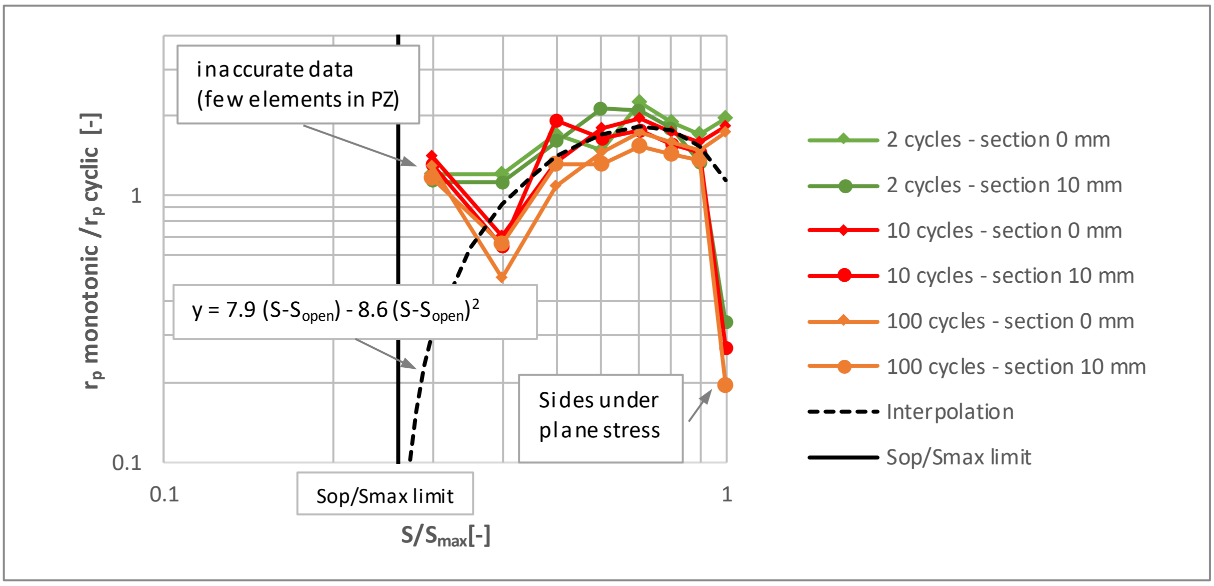


Fig.4 The ratio of $r_{p\,monotonic\,RD}/r_{p\,cyclic}$ based on the FE-analysis and the interpolation function for $B/r_{p\,max} = 79$

The interpolated function yielding the ratio of the monotonic plastic zone size $r_{p\,monotonic\,RD}$ under plane strain conditions versus the cyclic plastic zone size $r_{p\,cyclic}$ is proposed by Eq. 2.

The values of plastic zone size ratio close to the opening load level are influenced by the mesh size and are not assumed. The formula estimates the cyclic plastic zone size $r_{p\,cyclic}$ at each load level in upward part of the cycle for $R = 0$.

$$\frac{r_{p\,monotonic\,RD}}{r_{p\,cyclic}} = A\left(S - S_{open}\right) + B\left(S - S_{open}\right)^2 \qquad \text{for} \qquad S \in \left(S_{open}, S_{\max}\right) \tag{2}$$

The constants A = 7.9 and B = - 8.6 were determined for specimen´s characteristic dimension ratio $B / r_{p\max} = 79$. The ratio of plastic zone sizes $r_{p\,monotonic\,RD} / r_{p\,cyclic}$ is increasing along with the loading from S_{open} up to 0.7 $S_{\max}$ and then slightly decreases in the case of studied configuration with $B / r_{p\max} = 79$ and $R = 0$. The plastic zone size for $S / S_{\max} = 1$ depends on the position; i.e. at the sides or in the inner location. Evaluating the plastic zone size at the sides to the most remote plastically deformed element yields the rate of $r_{p\,monotonic\,RD} / r_{p\,cyclic} \approx 1/4$, but measuring continuous plastic zone yields $r_{p\,monotonic\,RD} / r_{p\,cyclic} \approx 3/2$.

Conclusions

The development of the plastic zone in M(T) specimen during cyclic loading is analysed in the 1st, 2nd, 10th and the 100th cycle using finite elements. The size and the shape are reported in the centre and at the sides of the specimen.

After the 1st monotonic cycle, the cyclic plastic zone is created. Its size in the centre of the specimen, where plane strain prevails, is approximately constant, but near the free sides, the zone disintegrates along with increasing number of cycles and tends to form the shape and size under plane strain conditions. The development of the plastic zone in upward part of the cycle can be described by power law. The coefficients of power law are summarized in Tab. 1 for the cross-section areas development and for the zone lengths as well.

Tab.1 Power law coefficients of plastic zone development ($B / r_{p\max} = 79$)

$y = C\left(\dfrac{S - S_{open}}{S_{\max} - S_{open}}\right)^k$		Monotonic plastic zone		Cyclic plastic zone	
		C [-]	k [-]	C [-]	k [-]
y = Cross – section area	Plane strain (centre of the specimen)	0.116	4.3	0.003	1.4
	Plane stress (specimens sides)				
y = Plastic zone size r_p	Plane strain (centre of the specimen)	0.040	1.8	0.019	0.7
	Plane stress (specimens sides)	0.124	3.0		

The ratio of monotonic plastic zone size versus cyclic plastic zone size $r_{p\,monotonic\,RD} / r_{p\,cyclic}$ can be described by interpolation function with the extreme between S_{open} and $S_{\max}$ and does not change significantly during cyclic loading except the maximum load level. On the sides at the maximum load level $S_{\max}$ the ratio $r_{p\,monotonic\,RD} / r_{p\,cyclic}$ is increasing due to the decreasing plastic zone size along with applied cycles. This behaviour indicates that the non-monotonic phenomenon appears. The development of plastic zone size $r_{p\,cyclic}$ according to monotonic plastic zone size contains the extreme value inside the range $\left(S_{open}, S_{\max}\right)$. At the sides, where plane stress condition prevails the ratio $r_{p\ monotonic\ RD} / r_{p\ cyclic} \approx 1/4$ was determined. As the plastic zone disintegrates after several cycles applied the ratio changes to approximately 3/2 at the load level $S_{\max}$.

The results obtained by presented analysis are determined under several constraints and so further analyses should follow. The description of plastic zone size development during cyclic loading can be utilized for the estimation of its size based on the monotonic plastic zone size derived according to analytical formulas or FE-analysis.

Acknowledgement

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References

[1] J. Kunz, Applied Fracture Mechanics, Czech Technique Association – CTU publishing (originally: Aplikovaná Lomová Mechanika, Česká Technika – nakladatelství ČVUT), (2005).

[2] J. Šedek, Numerical Investigation of 3-D Strain Constraint in Lab Test Specimens, Trebuňa F. et al., (eds.). EAN 2017 - 55th Conference on Experimental Stress Analysis 2017: Conference Proceedings, May 30th – June 1st, 2017, Nový Smokovec, Slovakia, TU Košice – Faculty of ME, (2017) 760 pages, pp. 174-177; ISBN 978-80-553-3167-6.

[3] G.R Irwin, Plastic Zone Near a Crack and Fracture Toughness, Proc. 7th Sagamore, (1960) conf. p. IV-63.

[4] D.S. Dugdale, Yielding of Steel Sheets Containing Slits, J. Mech. Phys. Solids 8, (1960) pp. 100 – 104.

[5] JC Newman Jr., A Crack Opening Stress Equation for Fatigue Crack Growth, International Journal of Fracture, Vol. 24, No. 3, (1984) pp. R131 - R135.