Multiobjective Optimization by Microgenetic Algorithm

Evolutionary algorithms are commonly used in optimization. They are robust, the design variables and the objective space may be discontinuous. Under some circumstances, they can be global. On the other hand, their exact convergence is slow, and are highly sensitive to little setup-changes, premature convergence.

There are many evolutionary algoritms: **ant colony algorithm**, **particle swarm algorithm**, **simulated annealing** and others.

Genetic algoritms have gained much popularity in the course of years. They are mimicking the natural evolution.

The basic genetic operators:

Selection
Crossover
Mutation

Most of real-world problems are of conflicting objectives. GA's were used originally for single-objective optimization, however they are well suited for multicriteria-tasks too.

The weighted sum method. Its usage is very simple (problem transformed into single-objective), fast. Potential drawbacks: concave Pareto-fronts and difficulties with the diversity of solutions.

Genuine multi-objective algorithms can tackle multi-objective tasks directly, with no problems with concave fronts. It is possible to get the whole trade-off surface among objectives in a single run.

Potential drawback: Interpretation of the results becomes an issue when the number of objectives is high.

In order to reduce the computational costs as much as possible, the concept of ***micro-genetic*** algorithm was introduced by **K. Krishnakumar**. It was based on the observation of **D. Goldberg** that a population of merely three individuals is necessary to get convergent evolution. Krishnakumar's algorithm has proven to be much faster, than its macro-variant.

His approach utilized a population of only five individuals, tournament selection, crossover and reinitialization instead of mutation.

C. A. Coello, G.T. Pulido applied microevolution to multi-objective optimization. Their **micro-GA** showed very fast convergence on several test examples.

In our approach, the micro-evolution was extended by **range adaptation** and reinitialization based on **sub-archives**.

εμARMOGA

(Adaptive-Range Multi-Objective Micro-genetic algorithm with ε-dominance)
It was tested extensively on a set of test-functions, exemplifying various challenges.
More details can be found in our paper accepted for publication in ***Vol. 40, No.6, June 2009, pp. 419-430, Advances in Engineering Software, Elsevier Sciences.***

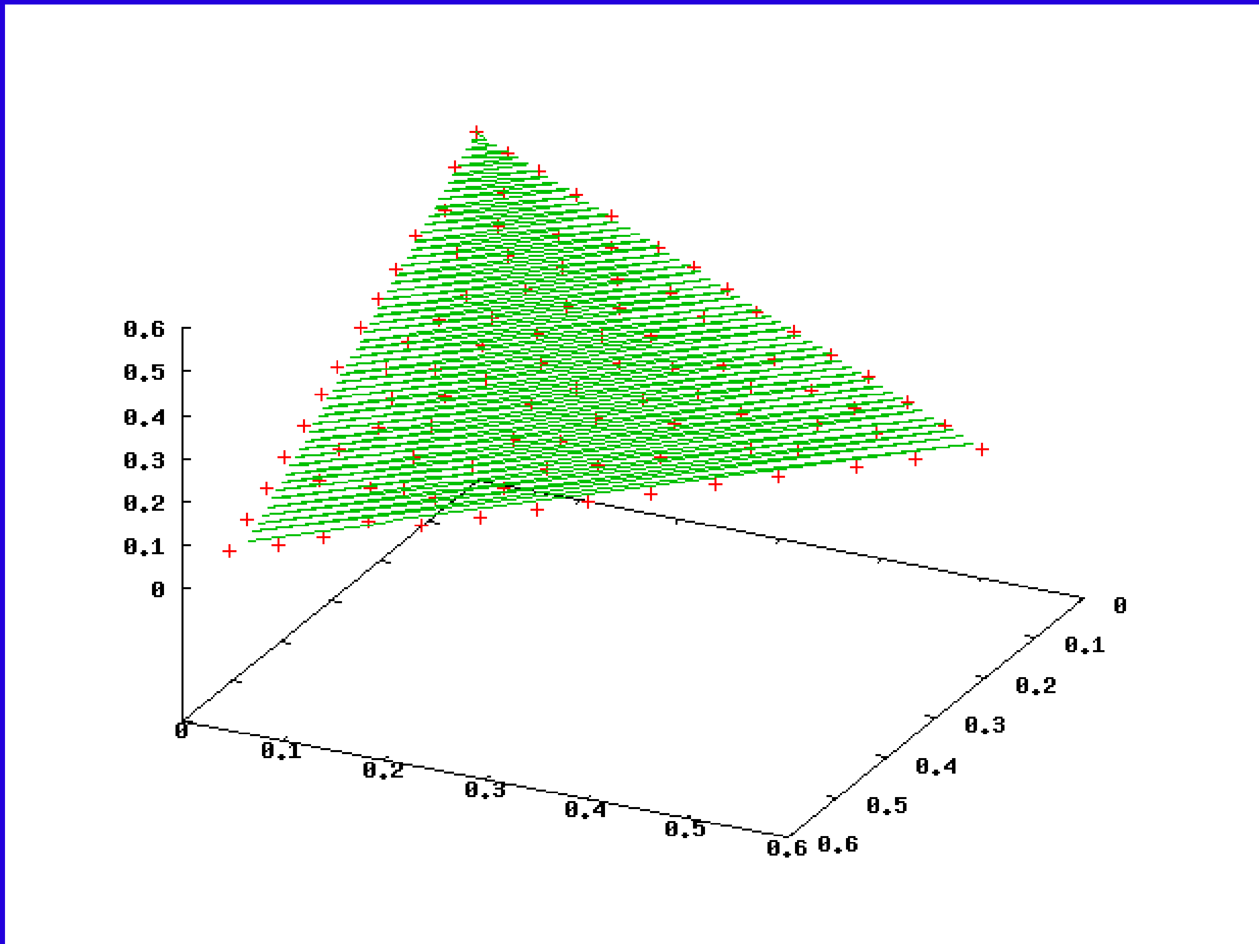
The setup of εμARMOGA:

Population size: 4
Selection method: tournament
Crossover scheme: one-point
Number of mated individuals: 4
No mutation
Archive size: 100
Adaptation parameter: 1.4
Minimum-sigma: 0.005

The maximum-number of objective-function evaluations was set to 20000 for two-objective problems and to 80000 for the problem DTLZ1.

The algorithm performed quite well on the two-objective test functions even when the number of function evaluations was limited to 2000.

Test function DTLZ1



Real-World Example
Hybrid optimization of airfoils

Because many types of airplanes have to fly also in adverse weather conditions such as rain contamination, pollution by insects or even beginning of icing, we decided to optimize airfoils simultaneously with both natural and forced transition into turbulent boundary layer. This resulted in a nontrivial ten-objective optimization problem, which we termed as hybrid optimization.

Six evolutionary objectives:

For both regimes:

Minimum-cruising drag coefficient
Minimum-manoeuvering drag coefficient
Maximum-lift coefficient

Four constraints:

Minimum airfoil thicknes
Two minimum-pitching moments
Minimum-thickness of the trailing edge

Problem formulation:

As the baseline design, the MS(1)-0317 - airfoil was chosen.

Minimum-thickness of the airfoil: 17% of the chord between 35% and 45% of the chord.
Minimum-thickness of the trailing-edge about 0.7%.
Minimum pitching moment > -0.07
Maximum-lift coefficient > 2.0 (for Re = 4000000, M = 0.14)

Performance of the baseline design:

Laminar regime:

Min. cruising drag coefficient.: 0.00644
Min. manoeuvering drag coefficient.: 0.00765
Max. lift coefficient: 2.0163
Pitching moment: -0.0757

Turbulent regime:

Min. cruising drag coefficient.: 0.00901
Min. manoeuvering drag coef.: 0.01023
Max. lift coefficient: 1.9749
Pitching moment: -0.0736

Our own **GPARSEC**-functions were used as a means for parameterization of the airfoils, and the **XFoil**-package was utilized for their evaluation.

Setup of εμARMOGA:

Population size: 10
Selection method: tournament
Crossover scheme: one-point
Number of mated individuals: 10
Mutation probability: 0.1
Mutation rate: 0.1
Archive size: 1000
Adaptation parameter: 1.2
Minimum-sigma: 0.4
Max. number of airfoil evaluations: 20000

Fig. 1: Laminar Projection of the Pareto Front – General View

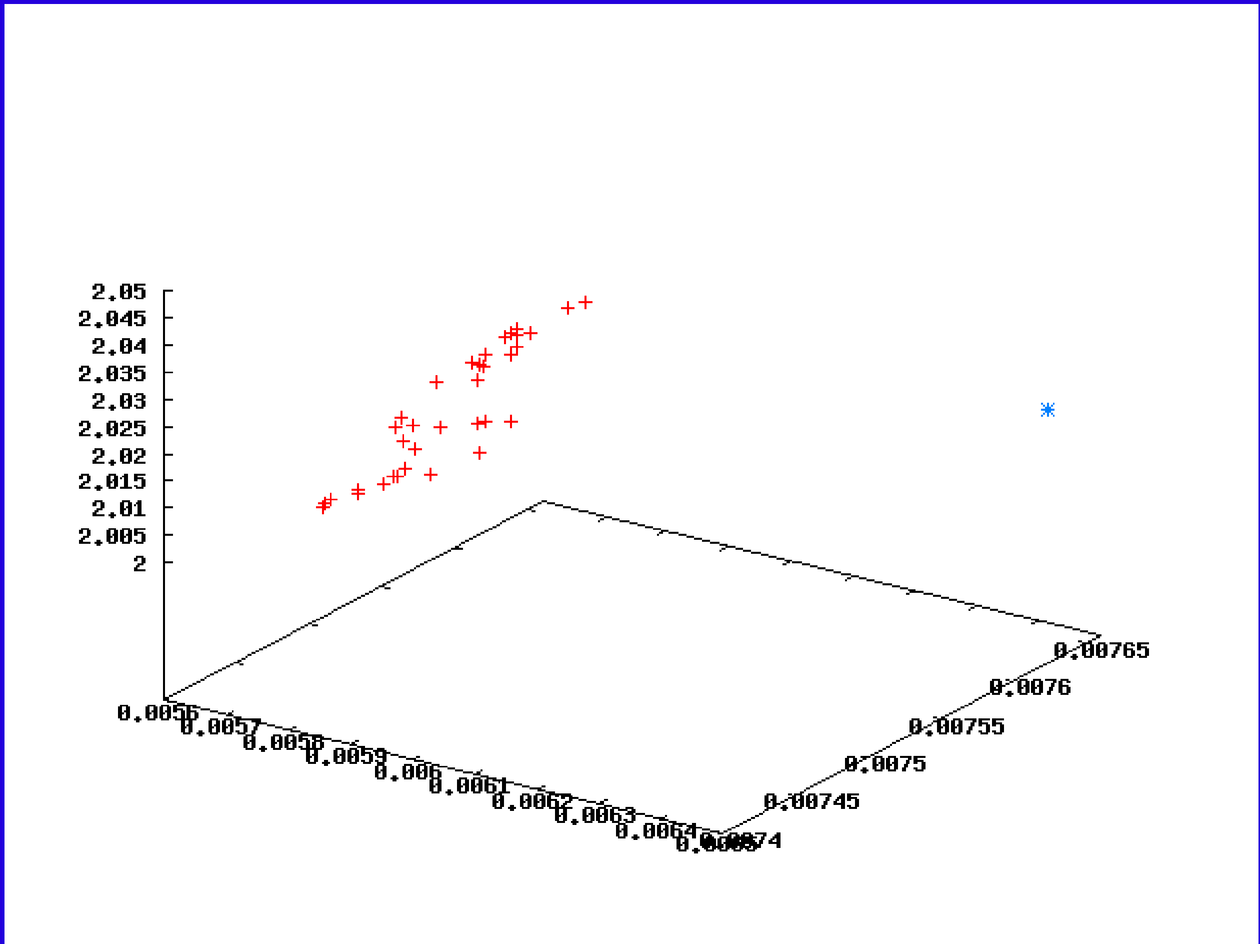


Fig. 2: Min. Cruising Drag vs. Max. Lift Coefficient

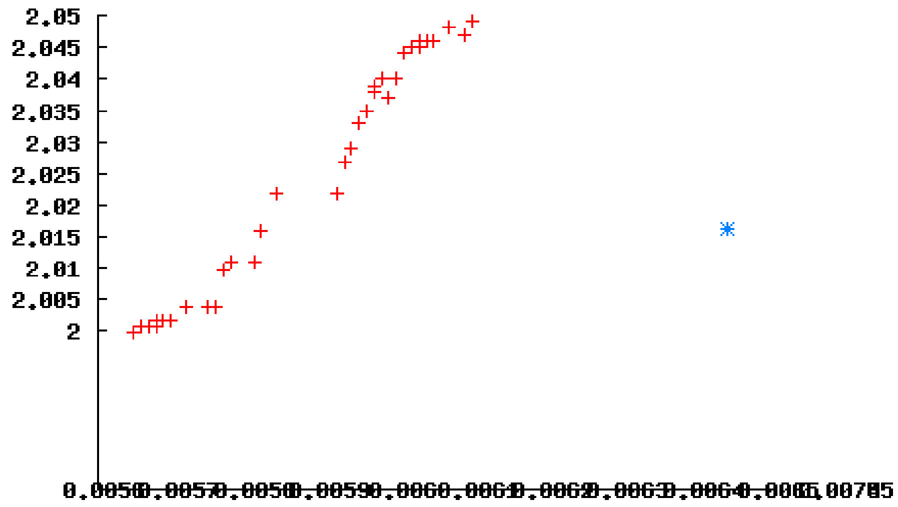


Fig. 3: Min. Cruising Drag vs. Min. Manoeuvering Drag

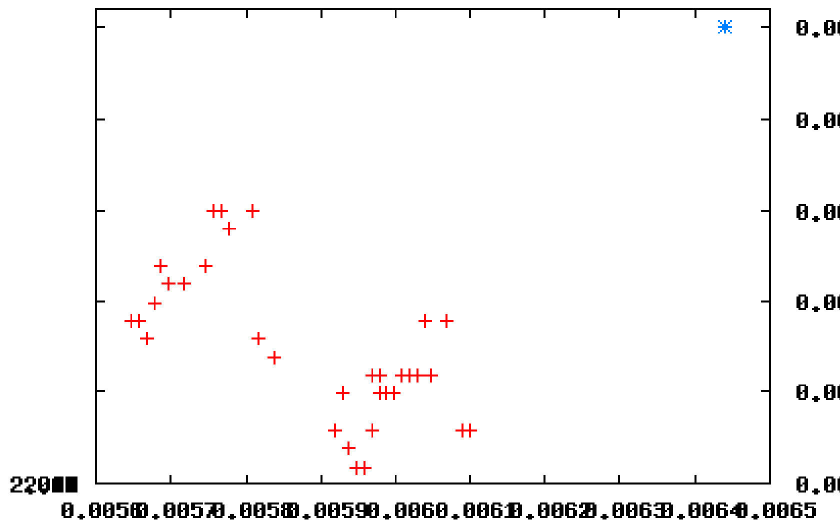


Fig. 4: Turbulent Projection of the Pareto Front – General View

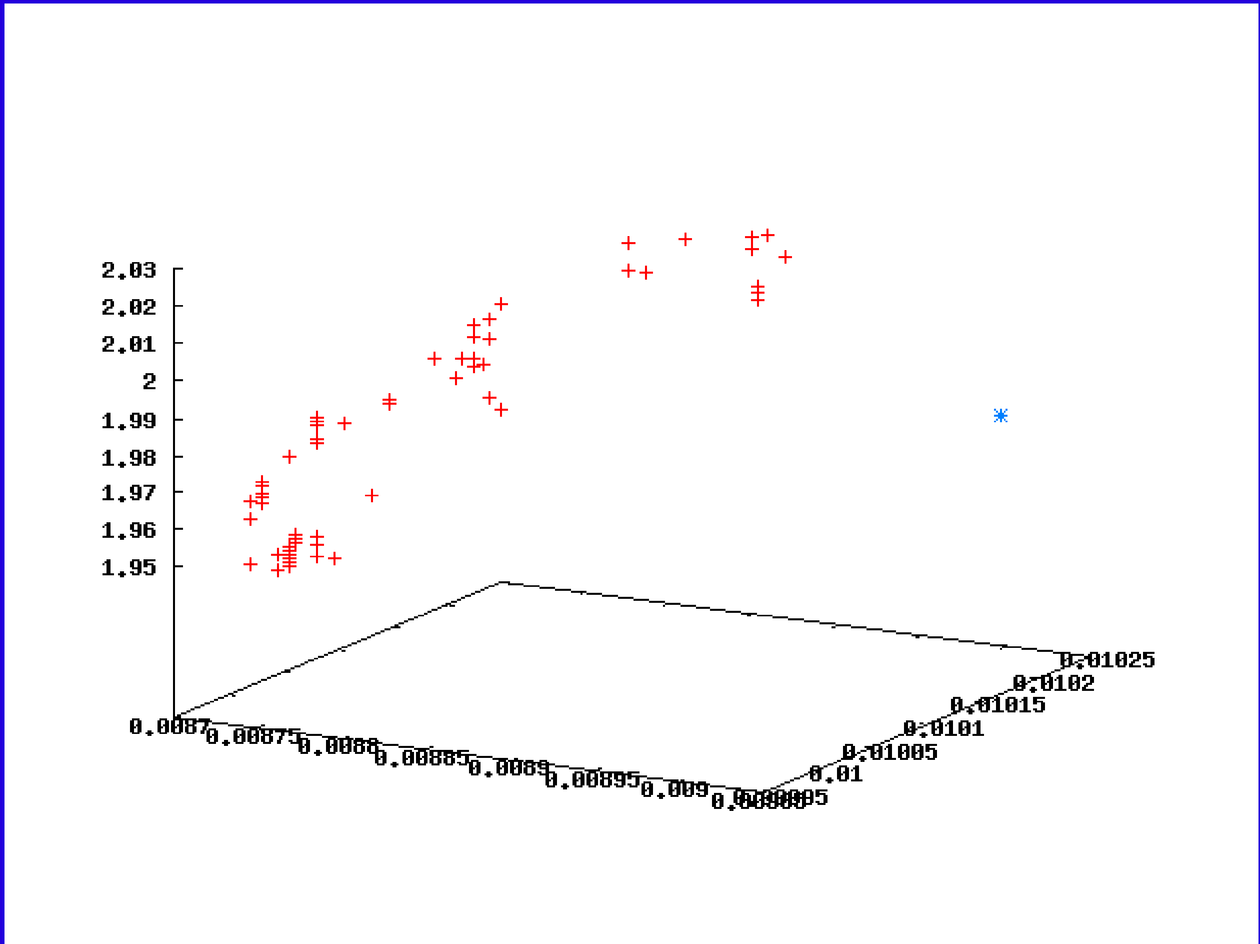


Fig. 5: Min. Cruising Drag vs. Max. Lift Coefficient

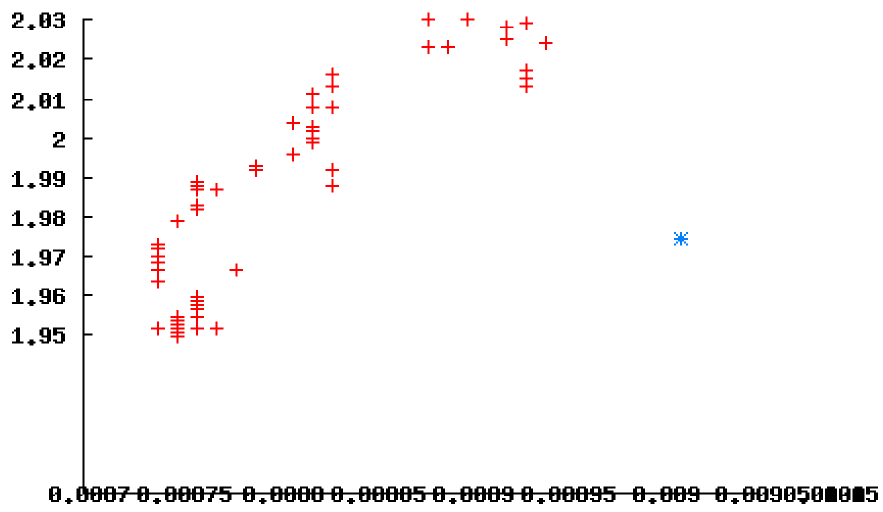
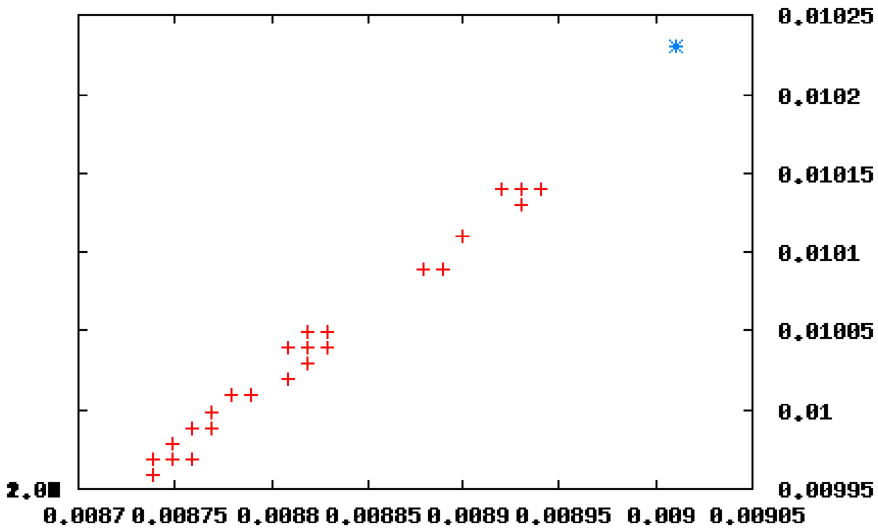


Fig. 6: Min. Cruising Drag vs. Min. Manoeuvering Drag



Results

We were able to supersede the baseline design by approximately 11% in laminar min. cruising drag coefficient, and by 5% in laminar maneouvering drag coefficient.
In turbulent regime, we have got up to 3%-gain for both drag-coefficients.
The maximum-gain for the lift-coefficient went up to 3% in both regimes.
All this when taking into account that both max. lift coefficients should be above or equal 2.

Conclusions:

Using of very small (micro) populations in evolutionary optimizations is a viable alternative. It can lead to considerable reduction of computation costs by quick convergence of the evolution to the vicinity of the global Pareto-front – the evolution produced excellent airfoils after merely few hundred airfoil-evaluations (15 minutes of computation on our hardware) in some cases of interest.

This fully justifies the usage of our **εμARMOGA** for multi-objective optimizations.